

ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION

MATHEMATICS

TIME – 3 HOURS

FULL MARKS : 100

- Note:** 1. Question No. 1 to 14 carry 4 marks each
2. Question No. 15 to 30 carry 5 marks each
3. Calculator is not allowed

SOLVE ANY TEN (10) QUESTIONS FROM QUESTION NO 1 TO 14

1. Prove that $5 - \sqrt{3}$ is an irrational number.
2. Show that $x = 2, y = 1$ and $x = 4, y = 4$ are solutions of the system of equations $3x - 2y = 4$ and $6x - 4y = 8$.
3. 4 chairs and 3 tables cost Rs. 2100 and 5 chairs and 2 tables cost Rs. 1750. Find the cost of a chair and a table separately.
4. If α, β are the zeros of the polynomial $f(x) = x^2 + x + 1$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.
5. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.
6. Express each of the following positive integers as the product of its prime factors: 3825 and 5005.
7. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.
8. Show that the points (1, -1), (5, 2) and (9, 5) are collinear.
9. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$.
10. A man goes 10m due east and then 24m due north. Find the distance from the starting point.
11. The diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC. Prove that $DF \times EF = FB \times FA$.
12. The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them saves Rs. 200 per month, find their monthly incomes.
13. Divide the polynomial $u(x) = 9x^4 - 4x^2 + 4$ by the polynomial $v(x) = 3x^2 + x + 1$. Also, find the quotient and remainder.
14. A dealer sells a toy for Rs. 24 and gains as much percent as the cost price of the toy. Find the cost price of the toy.

ATTEMPT ANY 12 (TWELVE) QUESTIONS FROM Q. 16 TO Q. 30.

15. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?
16. If an isosceles triangle ABC in which $AB = AC = 6$ cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

17. From the top of a hill, the angles of depression of two consecutive kilometer stones due east are found to be 30° and 45° . Find the height of the hill.
18. Find the coordinates of the circum-centre of the triangle whose vertices are $(8, 6)$, $(8, -2)$ and $(2, -2)$. Also, find its circum-radius.
19. Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10 km/hr. How much area will it irrigate 30 minutes, if 8 cm of standing water is required for irrigation?
20. A two digit number is four times the sum and three times the product of its digits. Find the number.
21. Draw the graph of the polynomial $f(x) = -4x^2 + 4x - 1$. Also, find the vertex of this parabola.
22. Show graphically that the system of equations $2x + 4y = 10$ and $3x + 6y = 12$ has no solution.
23. Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present age.
24. If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.
25. Solve the following quadratic equation by factorization method:

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, \quad a+b \neq 0.$$

26. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.
27. The sum of the first p, q, r terms of an A. P. are a, b, c respectively. Show that

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0.$$

28. The following data gives the distribution of total household expenditure (in rupees) of manual workers in a city:

Expenditure (in Rs.)	Frequency	Expenditure (in Rs.)	Frequency
1000-1500	24	3000-3500	30
1500-2000	40	3500-4000	22
2000-2500	33	4000-4500	16
2500-3000	28	4500-5000	7

Find the average expenditure which is being done by the maximum number of manual workers.

29. The cost of painting the total outside surface of a closed cylindrical oil tank at 60 paise per sq. dm is Rs. 237.60. The height of the tank is 6 times the radius of the base of the tank. Find its volume correct to two decimal places.
30. Find the area of the segment of a circle, given that the angle of the sector is 120° and the radius of the circle is 21 cm. (Take $\pi = 22/7$).